## ADVANCED GCE <br> MATHEMATICS (MEI)

Mechanics 3

Candidates answer on the Answer Booklet
OCR Supplied Materials:

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Wednesday 27 January 2010
Afternoon
Duration: 1 hour 30 minutes


## INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer all the questions.
- Do not write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $\mathrm{g} \mathrm{m} \mathrm{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g=9.8$.


## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.
- This document consists of 4 pages. Any blank pages are indicated.

1 (a) (i) Write down the dimensions of density, kinetic energy and power.
A sphere of radius $r$ is moved at constant velocity $v$ through a fluid.
(ii) In a viscous fluid, the power required is $6 \pi \eta r v^{2}$, where $\eta$ is the viscosity of the fluid.

Find the dimensions of viscosity.
(iii) In a non-viscous fluid, the power required is $k \rho^{\alpha} r^{\beta} v^{\gamma}$, where $\rho$ is the density of the fluid and $k$ is a dimensionless constant.

Use dimensional analysis to find $\alpha, \beta$ and $\gamma$.
[6]
(b) A rock of mass 5.5 kg is connected to a fixed point O by a light elastic rope with natural length 1.2 m . The rock is released from rest in a position 2 m vertically below O , and it next comes to instantaneous rest when it is 1.5 m vertically above O .

Find the stiffness of the rope.

2 (a) A uniform solid hemisphere of volume $\frac{2}{3} \pi a^{3}$ is formed by rotating the region bounded by the $x$-axis, the $y$-axis and the curve $y=\sqrt{a^{2}-x^{2}}$ for $0 \leqslant x \leqslant a$, through $2 \pi$ radians about the $x$-axis.

Show that the $x$-coordinate of the centre of mass of the hemisphere is $\frac{3}{8} a$.
(b) A uniform lamina is bounded by the $x$-axis, the line $x=1$, and the curve $y=2-\sqrt{x}$ for $1 \leqslant x \leqslant 4$. Its corners are $A(1,1), B(1,0)$ and $C(4,0)$.
(i) Find the coordinates of the centre of mass of the lamina.

The lamina is suspended with AB vertical and BC horizontal by light vertical strings attached to A and C, as shown in Fig. 2. The weight of the lamina is $W$.


Fig. 2
(ii) Find the tensions in the two strings in terms of $W$.

3 A particle P of mass 0.6 kg is connected to a fixed point O by a light inextensible string of length 1.25 m . When it is 1.25 m vertically below $\mathrm{O}, \mathrm{P}$ is set in motion with horizontal velocity $6 \mathrm{~m} \mathrm{~s}^{-1}$ and then moves in part of a vertical circle with centre $O$ and radius 1.25 m . When OP makes an angle $\theta$ with the downward vertical, the speed of P is $v \mathrm{~m} \mathrm{~s}^{-1}$, as shown in Fig. 3.1.


Fig. 3.1
(i) Show that $v^{2}=11.5+24.5 \cos \theta$.
(ii) Find the tension in the string in terms of $\theta$.
(iii) Find the speed of P at the instant when the string becomes slack.

A second light inextensible string, of length 0.35 m , is attached to P , and the other end of this string is attached to a point C which is 1.2 m vertically below O . The particle P now moves in a horizontal circle with centre C and radius 0.35 m , as shown in Fig. 3.2. The speed of P is $1.4 \mathrm{~m} \mathrm{~s}^{-1}$.


Fig. 3.2
(iv) Find the tension in the string OP and the tension in the string $C P$.

## [Question 4 is printed overleaf.]

4 Fig. 4 shows a smooth plane inclined at an angle of $30^{\circ}$ to the horizontal. Two fixed points A and $B$ on the plane are 4.55 m apart with B higher than A on a line of greatest slope. A particle P of mass 0.25 kg is in contact with the plane and is connected to A and to B by two light elastic strings. The string AP has natural length 1.5 m and modulus of elasticity 7.35 N ; the string BP has natural length 2.5 m and modulus of elasticity 7.35 N . The particle P moves along part of the line AB , with both strings taut throughout the motion.


Fig. 4
(i) Show that, when $\mathrm{AP}=1.55 \mathrm{~m}$, the acceleration of P is zero.
(ii) Taking $\mathrm{AP}=(1.55+x) \mathrm{m}$, write down the tension in the string AP, in terms of $x$, and show that the tension in the string BP is $(1.47-2.94 x) \mathrm{N}$.
(iii) Show that the motion of P is simple harmonic, and find its period.

The particle P is released from rest with $\mathrm{AP}=1.5 \mathrm{~m}$.
(iv) Find the time after release when P is first moving down the plane with speed $0.2 \mathrm{~m} \mathrm{~s}^{-1}$.

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