

# ADVANCED GCE MATHEMATICS (MEI)

4763

Mechanics 3

Candidates answer on the Answer Booklet

### **OCR Supplied Materials:**

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

## **Other Materials Required:**

None



Duration: 1 hour 30 minutes



#### **INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer all the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \, \text{m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use g = 9.8.

## **INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to
  indicate that a correct method is being used.
- The total number of marks for this paper is 72.
- This document consists of 4 pages. Any blank pages are indicated.

[4]

1 (a) (i) Write down the dimensions of density, kinetic energy and power. [3]

A sphere of radius r is moved at constant velocity v through a fluid.

- (ii) In a viscous fluid, the power required is  $6\pi\eta rv^2$ , where  $\eta$  is the viscosity of the fluid. Find the dimensions of viscosity.
- (iii) In a non-viscous fluid, the power required is  $k\rho^{\alpha}r^{\beta}v^{\gamma}$ , where  $\rho$  is the density of the fluid and k is a dimensionless constant.

Use dimensional analysis to find  $\alpha$ ,  $\beta$  and  $\gamma$ . [6]

(b) A rock of mass 5.5 kg is connected to a fixed point O by a light elastic rope with natural length 1.2 m. The rock is released from rest in a position 2 m vertically below O, and it next comes to instantaneous rest when it is 1.5 m vertically above O.

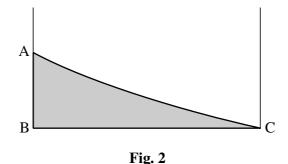
Find the stiffness of the rope. [6]

2 (a) A uniform solid hemisphere of volume  $\frac{2}{3}\pi a^3$  is formed by rotating the region bounded by the x-axis, the y-axis and the curve  $y = \sqrt{a^2 - x^2}$  for  $0 \le x \le a$ , through  $2\pi$  radians about the x-axis.

Show that the x-coordinate of the centre of mass of the hemisphere is  $\frac{3}{8}a$ . [5]

- (b) A uniform lamina is bounded by the *x*-axis, the line x = 1, and the curve  $y = 2 \sqrt{x}$  for  $1 \le x \le 4$ . Its corners are A (1, 1), B (1, 0) and C (4, 0).
  - (i) Find the coordinates of the centre of mass of the lamina. [9]

The lamina is suspended with AB vertical and BC horizontal by light vertical strings attached to A and C, as shown in Fig. 2. The weight of the lamina is W.



(ii) Find the tensions in the two strings in terms of W.

© OCR 2010 4763 Jan10

A particle P of mass 0.6 kg is connected to a fixed point O by a light inextensible string of length 1.25 m. When it is 1.25 m vertically below O, P is set in motion with horizontal velocity 6 m s<sup>-1</sup> and then moves in part of a vertical circle with centre O and radius 1.25 m. When OP makes an angle  $\theta$  with the downward vertical, the speed of P is v m s<sup>-1</sup>, as shown in Fig. 3.1.

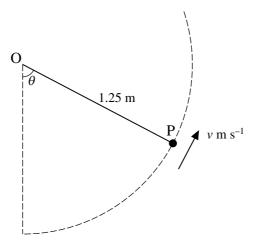


Fig. 3.1

- (i) Show that  $v^2 = 11.5 + 24.5 \cos \theta$ . [3]
- (ii) Find the tension in the string in terms of  $\theta$ . [4]
- (iii) Find the speed of P at the instant when the string becomes slack. [4]

A second light inextensible string, of length  $0.35 \, \text{m}$ , is attached to P, and the other end of this string is attached to a point C which is  $1.2 \, \text{m}$  vertically below O. The particle P now moves in a horizontal circle with centre C and radius  $0.35 \, \text{m}$ , as shown in Fig. 3.2. The speed of P is  $1.4 \, \text{m s}^{-1}$ .

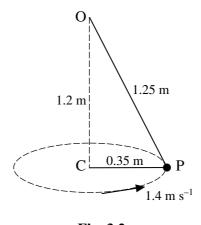


Fig. 3.2

(iv) Find the tension in the string OP and the tension in the string CP.

[7]

# [Question 4 is printed overleaf.]

[5]

4 Fig. 4 shows a smooth plane inclined at an angle of 30° to the horizontal. Two fixed points A and B on the plane are 4.55 m apart with B higher than A on a line of greatest slope. A particle P of mass 0.25 kg is in contact with the plane and is connected to A and to B by two light elastic strings. The string AP has natural length 1.5 m and modulus of elasticity 7.35 N; the string BP has natural length 2.5 m and modulus of elasticity 7.35 N. The particle P moves along part of the line AB, with both strings taut throughout the motion.

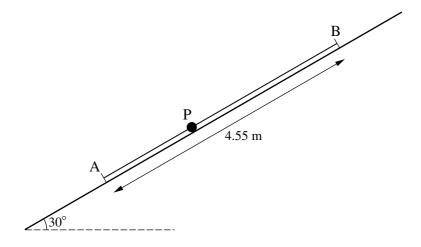


Fig. 4

- (i) Show that, when AP = 1.55 m, the acceleration of P is zero.
- (ii) Taking AP = (1.55 + x) m, write down the tension in the string AP, in terms of x, and show that the tension in the string BP is (1.47 2.94x) N. [3]
- (iii) Show that the motion of P is simple harmonic, and find its period. [5]

The particle P is released from rest with AP = 1.5 m.

(iv) Find the time after release when P is first moving down the plane with speed  $0.2 \,\mathrm{m \, s^{-1}}$ . [5]



#### Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations, is given to all schools that receive assessment material and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity. For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

© OCR 2010 4763 Jan10